

Contents

Preface	v
1 INTRODUCTION TO THE PROBABILITY THEORY	3
1.1 Definition of Probability	3
1.1.1 Set of elementary events	3
1.1.2 Axiomatic definition, probability space	7
1.1.3 Basic properties of probability	8
1.1.4 Relative-frequency definition	9
1.1.5 Classical definition	9
1.1.6 Geometrical definition	9
1.1.7 Conditional probabilities and independent events	9
1.1.8 Total probability	10
1.1.9 Bayes theorem	11
1.2 Distribution Functions	11
1.2.1 Random variable	11
1.2.2 Definition of distribution function	12
1.2.3 Properties of the distribution function	13
1.2.4 Density function	13
1.2.5 The distribution and density of function of one random variable	15
1.2.6 Random vectors	16
1.2.7 Marginal and conditional distributions	17
1.2.8 Joint distribution and density functions	18
1.3 Expected Values and Moments of Random Variables	20
1.3.1 Mathematical expectation	20
1.3.2 Variance	21
1.3.3 Properties of variance and correlation coefficients	22
1.3.4 Quantiles	24
1.3.5 Characteristics of a distribution density function	24
1.4 Characteristic and Generating Functions	27
1.4.1 Characteristic functions	27

1.4.2	Moment generating function	28
1.4.3	Semi-invariants or cumulants	29
1.5	Limit Theorems	29
1.5.1	Convergence in probability	29
1.5.2	Chebyshev inequality	30
1.5.3	Chebyshev theorem	30
1.5.4	Central-limit theorem	31
1.6	Discrete Distribution Functions	33
1.6.1	Binomial distribution	33
1.6.2	Poisson distribution	34
1.6.3	Geometrical distribution	35
1.7	Continuous Distribution Functions	36
1.7.1	Univariate normal distribution	36
1.7.2	Multivariate normal distribution	38
1.7.3	Uniform distribution	40
1.7.4	χ^2 -distribution	42
1.7.5	Student's t -distribution	43
1.7.6	Fisher distribution	44
1.7.7	Exponential distribution	45
1.7.8	Laplace distribution	46
1.7.9	Cauchy distribution	46
1.7.10	Logarithmic normal distribution	47
1.7.11	Significance of the normal distribution	47
1.7.12	Confidence interval	48
2	INFORMATION AND ENTROPY	53
2.1	Entropy of the Set of Discrete States of System	53
2.2	Entropy of the Complex System	53
2.3	Shannon Information (Discrete Case)	55
2.4	Entropy and Information (Continuous Case)	57
2.5	Fisher Information	61
3	RANDOM FUNCTIONS AND THEIR PROPERTIES	67
3.1	Expected Value and Variance	68
3.2	Properties of Random Functions	68
3.2.1	Stationarity	68
3.2.2	Ergodicity	69
3.3	Properties of Autocorrelation Functions	70
3.4	Action of a Linear Operator on Random Function	71
3.5	Cross-Correlation Function	72
3.6	Wiener-Khinchin Theorem and Power Spectrum	73
3.7	Examples of Correlation Functions and Power Spectrum	74
3.8	Estimation of Numerical Characteristics of Random Function ..	75
4	ELEMENTS OF MATHEMATICAL STATISTICS	79
4.1	Sampling and Estimation	79

4.2	Consistent Estimator	80
4.3	Unbiased Estimator	80
4.4	Rao-Kramer Inequality. Efficient Estimator	82
4.5	Sufficient Estimate	84
4.6	Robust Estimator	84
5	MODELS OF MEASUREMENT DATA	87
5.1	Additive Models	87
5.2	Models of Quantitative Interpretation	88
5.3	Regression Model	89
5.4	Models of Qualitative Interpretation	91
5.5	Models of Qualitative-Quantitative Interpretation	92
5.6	Random Component Models and Their Properties	93
5.6.1	The normal distribution	93
5.6.2	The random component is a stationary process	94
5.6.3	The random component is a non-correlated process ...	94
5.6.4	The random component is Markov process	94
5.6.5	The Laplace distribution	95
5.7	Model with Random Parameter	95
5.8	A Priori Information about Field of Parameters	96
6	SOLUTION OF THE INVERSE GEOPHYSICAL PROBLEMS: CASE OF THE LINEAR MODEL	101
6.1	Method of Least Squares (MLS)	101
6.1.1	Properties of the MLS-estimation	102
6.2	Method of Least Squares: Orthogonal Polynomials	103
6.3	Method of Least Squares with Constraints	106
6.4	Maximum Likelihood Method (MLM)	108
6.5	Method of Least Modulus	110
6.6	Huber Robust Method	112
6.7	Andrews Robust Method	112
6.8	Bayes Method and Statistical Regularization	115
6.8.1	The recursion algorithm of the maximum a posteriori probability method	118
6.8.2	Kalman filter	119
6.9	Singular Analysis and Method of Least Squares	121
6.9.1	Resolution matrix	125
6.10	The Method of Backus and Gilbert	127
7	SOLUTION OF THE INVERSE GEOPHYSICAL PROBLEMS: CASE OF THE NONLINEAR MODEL	133
7.1	Newton-Le Came Method	133
7.2	Method of Least Squares: Nonlinear Case	135
7.3	Method of Least Squares for Non-Stationary Model	137
7.4	Interval Estimation Method	138
7.5	Genetic Algorithms	140

7.5.1	Coding	140
7.5.2	Selection	141
7.5.3	Crossover	142
7.5.4	Mutation	144
7.5.5	Choice	144
8	STATISTICAL CRITERIA FOR CHOICE OF MODELS	149
8.1	Parametric Hypothesis Test	149
8.2	Criterion of a Posteriori Probability Ratio	150
8.3	The Signal Resolution Problem	154
9	RAY GEOPHYSICAL TOMOGRAPHY METHOD	171
9.1	Definition of Ray Tomography	171
9.2	Radon Transform	172
9.3	Algebraic Methods	173
9.4	Slowness and Absorption Coefficient Reconstruction	176
9.5	Radon Transform for Seismogram Processing	179
9.6	Hilbert Transform and Analytical Signal	182
10	DIFFRACTION TOMOGRAPHY METHOD	187
10.1	Diffraction Tomography in Scalar Case	187
10.2	Born Approximation for Seismic Trace Model	191
10.3	Diffraction Tomography for Vector Displacement Field	194
10.3.1	Model of measurement	194
10.3.2	Definition of tomography functionals	196
10.3.3	Tomography functional for scalar wave equation	197
10.3.4	Tomography functional for Lame equation	198
10.4	Reconstruction Algorithm	205
10.5	Analysis of Recovery Errors and Comparison of the Reconstruction Tomography Algorithm with Backus-Gilbert Method	208
11	MULTIFACTOR ANALYSIS AND PROCESSING OF TIME SERIES	215
11.1	Time Series and Multifactor Analysis	215
A	ABOUT MATLAB	225
A.1	MATLAB Command Window	225
A.1.1	Menu bar of the command window	226
A.1.2	Option file	226
A.1.3	Option edit	226
A.2	Some Operations with Matrices	226
A.2.1	The elementary operations	226
A.3	Division of Matrices	228
A.4	Updating Matrices	228
A.5	Symbol ":"	229
A.5.1	Creation of vectors	230

A.5.2	Indexation	231
A.6	The Conditional Operators and Loops	232
A.7	The Functions and M-Files	233
A.7.1	Functional M-files	233
A.7.2	Script-file	234
A.8	Creation of the Elementary Graphics	234
B	STATISTICAL METHODS AND TRANSFORMS	239
B.1	Examples of Numerical Simulation of Random Values	239
B.1.1	Exercises	240
B.2	Creation of Histogram	240
B.2.1	Exercises	240
B.3	Description of a Random Variable	241
B.3.1	Beta distribution	241
B.3.2	Binomial distribution	241
B.3.3	Cauchy distribution	243
B.3.4	Chi-square distribution	243
B.3.5	Exponential distribution	244
B.3.6	Fisher distribution	244
B.3.7	Gamma distribution	244
B.3.8	Gaussian distribution	244
B.3.9	Geometrical distribution	244
B.3.10	Hypergeometrical distribution	245
B.3.11	Laplace distribution	245
B.3.12	Logarithmic normal distribution	245
B.3.13	Poisson distribution	245
B.3.14	Snedecor distribution	245
B.3.15	Student distribution	246
B.3.16	Uniform distribution	247
B.3.17	Two dimensional normal distribution	246
B.3.18	Exercises	246
B.4	Computer Simulation of Random Values	246
B.4.1	Kolmogorov criterion	247
B.4.2	Exercises	248
B.5	Confidence Intervals	248
B.5.1	Exercises	248
B.6	The Time Series	248
B.6.1	Correlated and non-correlating processes	248
B.6.2	Auto-correlation and cross-correlation functions	248
B.6.3	Exercises	249
B.7	Transforms	249
B.7.1	Fourier transform	249
B.7.2	Wavelets	250
B.7.3	Multifactor analysis	251
B.7.4	Cepstral transform	253
B.7.5	Exercises	255

C DIRECT AND INVERSE PROBLEM SOLUTION	263
C.1 Computer Simulation of Gravitational Attraction	263
C.1.1 Gravitational attraction of a sphere	263
C.1.2 Gravitational attraction of a horizontal cylinder	263
C.2 Computer Simulation of Magnetic Induction	264
C.2.1 Magnetic induction of a dipole	264
C.2.2 Magnetic induction of a horizontal cylinder	265
C.3 Computer Simulation of Seismic Field	265
C.3.1 Exercises	266
C.4 Deconvolution by the Wiener Filter	268
C.4.1 Exercises	270
C.5 Quantitative Interpretation	271
C.5.1 Exercises	275
C.6 Qualitative Interpretation	275
C.6.1 Exercises	276
C.7 Diffraction Tomography	277
C.7.1 Exercises	279
References	285
Index	287