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Nonlinear Evolution of the Accumulation Processes of the Material for Formation of the Giant Planets in the Primeval Solar System

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1. Introduction

During processes of the bi-polar flow in the primeval solar system, the gas flow excites the density waves which are evolved generating the stationary mode with respect to the proto-sun. In the initial phase of the wave growth, the evolution takes place in the regime of the weak turbulence which has been proposed by Oya (in this issue). Because the studies are concentrated in the initial phase where weak turbulence theory can be applied, it is needed to follow the processes in the late phase when the evolution will enter into the strong nonlinear regime. In this regime we can trace the processes only by the computer simulations. In this paper, the model and algorithm of the computer simulation, for the non-linear stage of the evolution of the density waves are presented; the results indicate the revolution stage that is followed by the growth in the quasi-linear regime. A large growth at the position of the nodes where Titius-Bode's law is satisfied results in the solitary waves. For systematic growth in this nonlinear phase, we have considered the interaction mechanism how the outward flow processes are maintained through the dense stationary component of the clouds which became the seeds of the giant planets.

2. Model and basic equation

Model

As has been discussed in the paper of the accumulation of the material in the quasi-linear phase, the situation in the bi-polar flow in the primeval solar system is considered. In Fig. 1, we depict the average over concept of the bi-polar flow. There are two components in the structure as has been pointed out by recent observation that dust rich disc is dressed circulating the center star where the gas components make out flow with the bi-polar structure (Fig. 1). From the observation of the Doppler shift of the radio wave frequencies from the bi-polar flows, it has been clarified that the gas flow has velocity between 1km/s to 10km/s (Kawabata *et al.*, 1990, Private communication). The statistics of the presently observed bi-polar flows, shows that the stage of the bi-polar flow usually continues in the time interval of about 1×10^3 to 1×10^4 yrs.

As has been proposed in the sister paper (Oya, in the present issue, hereafter we cite this paper as Quasi-linear paper in the present issue: QPI), the origin of the bi-polar flow is assumed to be caused by the electromag-

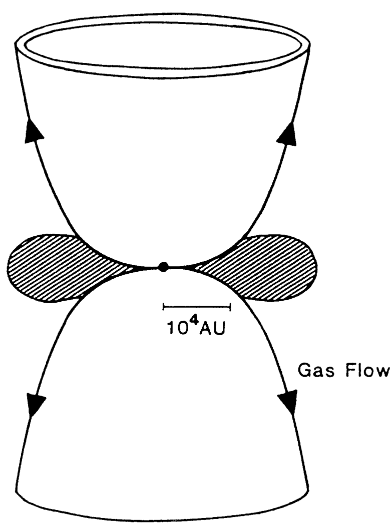


Fig. 1. Schematic configuration of the bi-polar flow corresponding to the early stage of the solar system. The dot at the center of the disc represents the proto-sun.

netic force in the rotating proto solarsphere with intense magnetic field. The plasma trapped in the rotating magnetic field flow out in the form of the wind caused by the centrifugal force. Because the plasma is dense, the plasma is recombined being neutralized in fairly early period of the flow out where the gas is sufficiently cooled down. The wind is therefore basically neutrarized gas flow with partially ionized components. In Fig. 1 of QPI paper, the model of the present study has been summarized. In the gas which is making outward flow from the central hot plasma region with rotating magnetic field, the disc region is formed.

Equations

The governing equations of the gas dynamics in the disc region are basically the same with QPI paper. We repeat the equation here because the modifications has been made for the computer simulations in the present paper. Again under the restriction of the ring model, the equation of the continuity is written by

$$\frac{\partial N_a}{\partial t} + \frac{\partial N_b}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rN_aV_a) = 0, \tag{1}$$

where N_a and N_b denote the gas densities of the flowing out and stationary components, respectively. For the equilibrium state at the initial stage, the continuum relation gives

$$\frac{\partial}{\partial r}(rN_aV_a) = 0. \tag{2}$$

This gives the solution

$$rN_{a0}V_a = C \tag{3}$$

for constant C which leads to

$$N_{a0} = \left(\frac{r_0}{r}\right) N_{00} \tag{4}$$

with $C = (r_0 V_a) N_{00}$.

The density makes evolution starting from the value given by Eq. (4). The equation of dynamics is written by

$$\begin{aligned} m_0 N \frac{\partial \boldsymbol{V}}{\partial t} \quad & -m_0 N (\boldsymbol{V} \times \operatorname{rot} \boldsymbol{V}) + \frac{1}{2} m_0 N \nabla V^2 \\ & + m_0 N \nabla \phi + \nabla (N \kappa T) + \eta \nabla^2 \boldsymbol{V} = 0, \end{aligned} \tag{5}$$

where m_0, N, $\boldsymbol{V}$, κ, T, and ϕ are the average mass of the element particles, number density, azimuthal velocity component, Boltzmann constant, temperature, and potential, respectively. In Eq. (5), η is the viscosity of the fluid; The density N is expressed being separated into the two components

$$N = N_a + N_b, \tag{6}$$

where N_a and N_b are the moving wave and stationary wave components, respectively. The potential ϕ is expressed by

$$\phi = \phi_0 + \phi_1 \tag{7}$$

with

$$\phi_0 = -G \frac{M_\odot^*}{r}, \tag{8}$$

where G and $M_\odot^*$ denote the gravitational constant and the total mass of the proto-sun, respectively. The perturbation of the gravity is resulted by self gravity force due to the concentration of the local density as

$$\nabla^2 \phi_1 = 4\pi G m_0 (N_a + N_b). \tag{9}$$

The continuity equation is expressed for three different density components

$$N = N_{a0} + N_{a1} + N_b \tag{10}$$

as

$$\frac{\partial(N_{a1} + N_b)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\{rV_{r1}N + rV_{r0}(N_{a1} + N_b)\} = 0 \tag{11}$$

with

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_{r0}N_{a0}) = 0,$$

where N_{a0}, N_{a1}, and N_b denote the stationary ambient density, the oscillating density wave component, and the stationary component of the density with time dependence, respectively; V_{r0} and V_{r1} denote the stationary outward flow velocity and oscillating velocity component of the gas, respectively. Equation (5) can be separated into the stationary and fast varying components as

$$N_{a0}m_0\left\{\frac{1}{2}(V_{r0}^2 + V_{\theta 0}^2)\right\} + \phi_0 + \kappa T N_{a0} = C \tag{12}$$

and

$$Nm_0\left\{\frac{\partial V_{r1}}{\partial t} + \frac{\partial}{\partial r}(V_{r0}V_{r1} + V_{r1}^2) + \frac{\partial \phi_1}{\partial r}\right\} + \kappa T\frac{\partial N_{a1}}{\partial r} = 0, \tag{13}$$

where the potential ϕ in Eq.(5) is separated as given in Eqs. (7) to (9). The density N_0 is separated by averaging from N_{a1} component as

$$\left\langle \frac{\partial N_b}{\partial t} \right\rangle = -\frac{1}{T}\int_0^T \frac{1}{r}\frac{\partial}{\partial r}\{rV_{r1}N + rV_{r0}(N_{a1}+N_b)\}dt. \tag{14}$$

In the above the integration time T is selected to be in a range

$$\tau_a < T < \tau_b, \tag{15}$$

where τ_a and τ_b are the characteristic times of the N_{a1} and N_b variations.

Assumption for z-component

In the treatment, we have applied a simple assumption to separate z-component from r- and θ-components. The Poisson equation corresponding to Eq.(9) can be rewritten in the present ring model as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_1}{\partial r}\right) + \frac{\partial^2 \phi_1}{\partial z^2} = 4\pi G\rho, \tag{16}$$

where

$$\rho = (N_a + N_b)m_0.$$

In the z-direction the balance condition gives a relation

$$\rho\frac{\partial \phi}{\partial z} = -\frac{\kappa T}{m_0}\frac{\partial \rho}{\partial z}. \tag{17}$$

This equation can be separated into the equilibrium and the time dependent components as

$$\rho_0\frac{\partial \phi_0}{\partial z} = -\frac{\kappa T}{m_0}\frac{\partial \rho_0}{\partial z} \tag{18}$$

and

$$\rho_1 \frac{\partial \phi_1}{\partial z} + \rho_0 \frac{\partial \phi_1}{\partial z} = -\left(\frac{\kappa T}{m_0}\right) \frac{\partial \rho_1}{\partial z}. \tag{19}$$

In this simulation, we have used the following model:

$$\rho_1 = \rho_1(r,t) f(z) \tag{20}$$

with

$$f(z) = \exp\left\{-\frac{GM_\odot^*}{2r^3}\left(\frac{m_0}{\kappa T}\right) z^2\right\}. \tag{21}$$

Because ϕ_0 and ρ_0 are given, respectively, by

$$\phi_0 = \frac{GM_\odot^*}{\sqrt{r^2 + z^2}} \tag{22}$$

and

$$\rho_0 = \rho_0(r) f(z). \tag{23}$$

It follows from Eq. (19) that

$$\frac{\partial \phi_1}{\partial z} = 0. \tag{24}$$

Then, we can rewrite Eq. (16) as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_1}{\partial r}\right) = 4\pi G\rho_1(r,t), \tag{25}$$

where

$$\rho_1(r,t) = m_0(N_a + N_b).$$

3. Scheme of the computer simulation and initial and boundary conditions

To carry out the computer simulation all basic equations are rewritten in normalized forms by chosing the following quantities:

$$\left.\begin{aligned} N^* &= N/N_{00}, \\ N_a^* &= N_a/N_{00}, \\ N_b^* &= N_b/N_{00}, \\ r^* &= r/r_0, \\ t^* &= t/(r_0/V_{r0}), \\ V_{r1}^* &= V_{r1}/V_{r0}, \\ \phi_1^* &= m_0\phi_1/\kappa T, \\ V_\theta^* &= V_{\theta 0}/V_{r0}, \\ k^* &= k/k_0. \end{aligned}\right\} \tag{26}$$

In Eq.(26), N_{00}, r_0, V_{r0}, and T are defined already relating to Eqs.(4), (5) and (11); we do not then repeat interpretations here. Using the normalized quantity, then we can rewrite all of the basic equations. We can, respectively, rewrite Eqs. (11) to (13), as Eqs. (27) to (29):

$$\frac{\partial(N_a^* + N_b^*)}{\partial t} + \frac{1}{r^*}\frac{\partial}{\partial r^*}\{r^* V_{r1}^* N^* + r^*(N_{a1}^* + N_b^*)\} = 0, \tag{27}$$

$$\left(\frac{V_{r0}}{V_{th}}\right)^2 (1 + V_\theta^{*2}) - \frac{1}{2}\left(\frac{V_G}{V_{th}}\right)^2 \frac{1}{r^*} + 1 = C, \tag{28}$$

and

$$\begin{aligned} &\left(\frac{V_{r0}}{V_{th}}\right)^2 (N_a^* + N_b^*)\left\{\frac{\partial V_{r1}^*}{\partial t^*} + \frac{\partial}{\partial r^*}(V_{r1}^* + V_{r1}^{*2})\right\} \\ &\quad + \frac{\partial \phi_1^*}{\partial r^*} + \frac{\partial(N_{a1}^* + N_b^*)}{\partial r^*} = 0, \end{aligned} \tag{29}$$

where $V_{th} = \sqrt{\kappa T/m_0}$. Equation (25) can be rewritten as

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \phi_1^*}{\partial r^*}\right) = \frac{3}{2}\left(\frac{V_G}{V_{th}}\right)^2 \left(\frac{N^*}{N_\odot^*}\right), \tag{30}$$

where

$$N_\odot^* = \frac{M_\odot^*/m_0}{\frac{4}{3}\pi r_0^3 N_{00}}, \tag{31}$$

$$N^* = N_a^* + N_b^*, \tag{32}$$

and

$$V_G = \sqrt{\frac{2GM_\odot^*}{r_0}}. \tag{33}$$

The computer simulation has been carried out applying two- step Lax-Wendroff method or a chain of the partial differential equations, with initial value at $t^* = 0$ as

$$N_{a1}^* = N_{a0}^*/10, \tag{34}$$

where $N_{a0} = 1/r^*$. In Eq.(27) the initial perturbation is given by

$$V_{r1}^* = 0. \tag{35}$$

For N_a^* and N_b^*, Eqs. (11) gives the evolution with Eq.(13).

As initial condition, perturbation is given to N_b^* in two ways; the first is Titius-Bode's law as

$$N_b^* = \alpha N_{a0}^* \sin\{k^* \ln r^*\} \tag{36}$$

with $\alpha = 1 \times 10^{-3}$ where $k_0 = 2\pi/r_0$.

The second case of the perturbation is the cluster of the four components of the waves with wave numbers (r_0/r_J), $k(r_0/r_S)$, $k(r_0/r_U)$, and $k(r_0/r_N)$, where r_J, r_S, r_U, and r_N are the distance of Jupiter, Saturn, Uranus, and Neptune, respectively. That is

$$N_b^* = \sum_{i=J}^{N} b^* \cos\left\{2\pi(\frac{r^*}{r_i^*}) + \varphi_i\right\} \tag{37}$$

with $b^* = 1 \times 10^{-3}$.

4. Results of simulations

The results for the initial condition given by Eq. (36), are given in Figs. 2(a) to 2(e) sequentially for the computer time $t_{cu} = 0,\ 2.1 \times 10^7$, 4.2×10^7, 6.3×10^7, and 8.4×10^7. These correspond to the real times of 2.5×10^7s, 5.0×10^7s, 7.5×10^7s, and 1.0×10^8s, respectively, where we select $r_0 = 4.0$AU that gives the first peak at the position of the present Jupiter. The results show the following feature: The quasi-stationary perturbation which starts to grow in the initial phase enters into the states of completely stationary perturbation after $t_{cu} = 6.3 \times 10^7$. Within 3yrs (for $r_0 = 4$AU, $V_{r0} = 5$km/s) then there appear the seeds of the material accumulation for the origin of giant planets.

For tenuous density of the gas, the evolution becomes slower. In Figs. 3a to 3e time sequence of the evolution for the case of $N_0 = 2 \times 10^{11}$/cc at the position of Jupiter is given. The second case of the growth feature that starts with many phases as given in Eq. (37) has been investigated as the results are given in Figs. 4(a) to 4(n). Though the function of $b^*(t,r)\sin(\Theta r^* + \varphi)$ can not be expressed by a simple function as is the case of the initial condition given by Eq. (36), with simple logarithmic function, the evolution shows that preferable accumulations are taking place corresponding to the positions of giant planets. This selected accumulations becomes especially apparent after $t_{cu} = 1.47 \times 10^8$ corresponding to 1.75×10^8s (5.5yrs) for the case of $V_{r0} = 5$km/s, and $N_0 = 10^{12}/\text{cm}^3$.

Comparing the results of these three cases of the simulations, it is clarified that the accumulation function of the density waves is due to nonlinear coupling of the density waves, of N_1 with velocity perturbation V_{r1}, that has been predicted by the quasi linear theory as described in QPI paper for the process evolving in the disc. The growth of the perturbation is always resulted as inherent characteristics in the flow out gas where the stationary perturbation components are making growth. The resulted density fluctuations show the accumulations at the positions preferable to Titius-Bode's law; this selection of the positions does not depend on the feature of initial perturbations.

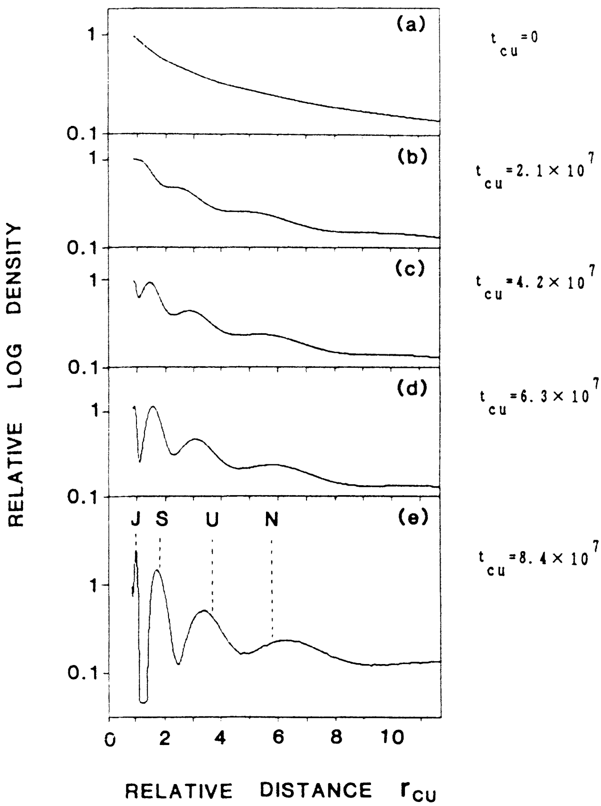


Fig. 2. The simulation results for evolution of the stationary component of the collected gas material for the case of the background gas number density of $1 \times 10^{12}/\mathrm{cm}^3$ at 4AU with radial out flow velocity of 5km/s. The results are plotted for the relative log density versus the distance. The time t_{cu} is normalized time given by the time unit of the simulation; 1 t_{cu} corresponds to 1.19 s in the present parameter of the calculation. J, S, U, and N show the present position of Jupiter, Saturn, Uranus, and Neptune, respectively. The simulation has been started with the density fluctuation of 10 % to the background density to satisfy Bode's law at $t_{cu} = 0$.

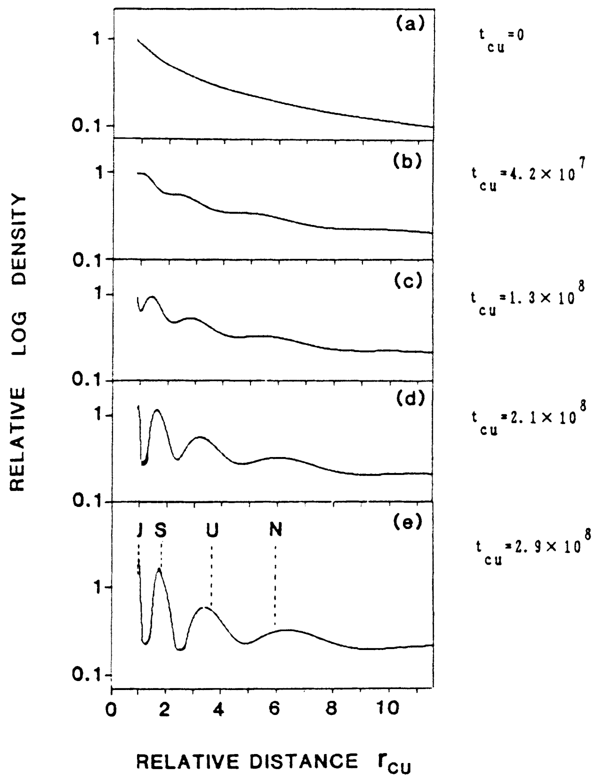


Fig. 3. Same as Fig. 2 but for the case of background gas number density of $2 \times 10^{11}/cm^3$ at 4AU.

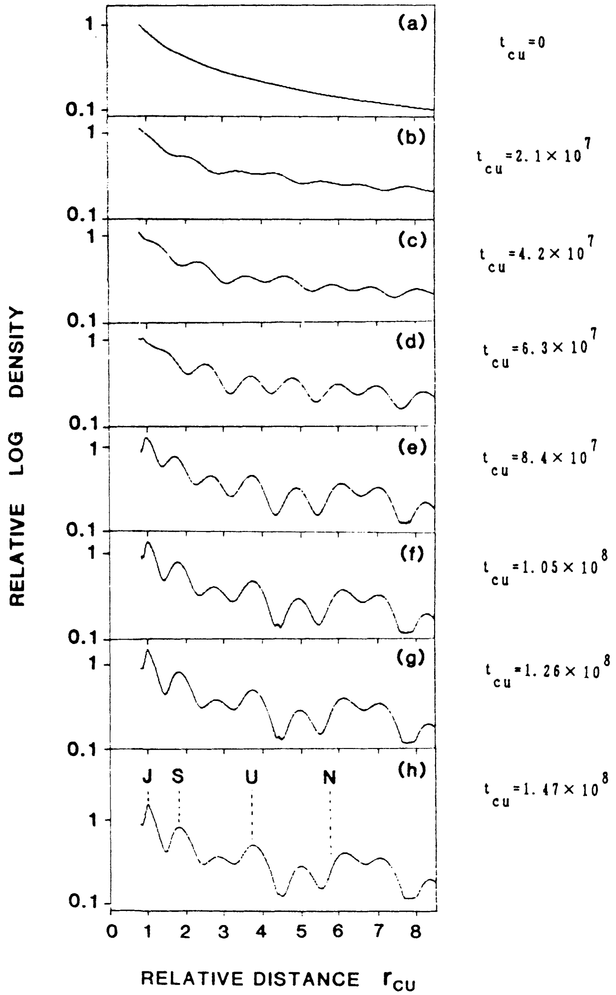
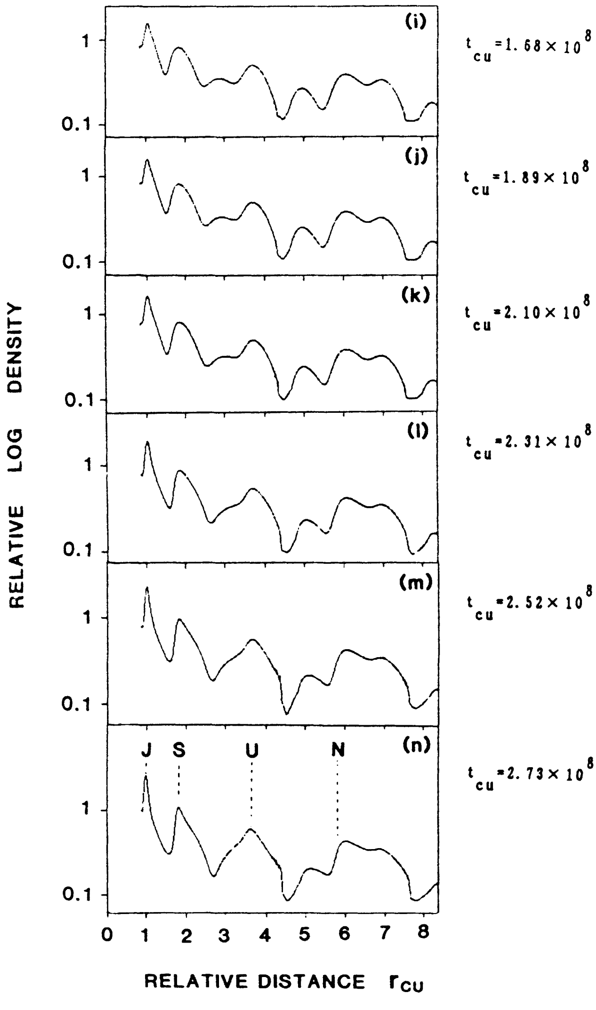


Fig. 4. The simulation results for evolution of the stationary component of the material for the case of $N_0 = 1 \times 10^{12}/\mathrm{cm}^3$ with the flow velocity $V_{r0} = 5\mathrm{km/s}$. The evolution from $t_{cu} = 0$ (a) to $t_{cu} = 2.73 \times 10^8$ (n) are given at each corresponding time



(1 t_{cu} = 1.19 s and t_{cu} = 2.73×10^8 corresponds to 3.25×10^8 s in the case of $1 \times 10^{12}/\mathrm{cm}^3$ for 5km/s), starting from the perturbation with four different k values (see main text).

5. Discussions: importance of the local electromagnetic effects

The present studies have been made for the number density ranging from 10^{11} to $10^{12}/\text{cm}^3$ with flow out velocity ranging from 5 km/s to 10 km/s. This gives the life time of the bi-polar flow stage from 1×10^5yrs to 5×10^5yrs which can be consistent with the results of statistics derived from the presently observed bi-polar flow cases. When the flow-out gas meet with the stationary components, the collision frequency becomes in a range $1 \sim 10/\text{s}$ assuming $10^{10} \sim 10^{11}$ for the number density of the stationary component in early stage of the accumulation of the material. The collision frequency is larger than characteristic time of the oscillation of the density waves. Therefore it is a critical issue to consider how the stationary component can persist without captured by the flowing out component of gas.

The essential solution to this problem is obtained by eddy motions of the stationary components; one of the possible eddy model is depicted in Fig. 5. In the axissymetric cylindrical coordinate system, the eddy $\boldsymbol{\omega} = \text{rot}\,\boldsymbol{V}$ can be expressed by

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \text{rot}(\boldsymbol{V} \times \boldsymbol{\omega}) + \nu \nabla^2 \boldsymbol{\omega}. \tag{38}$$

Then we can express, as

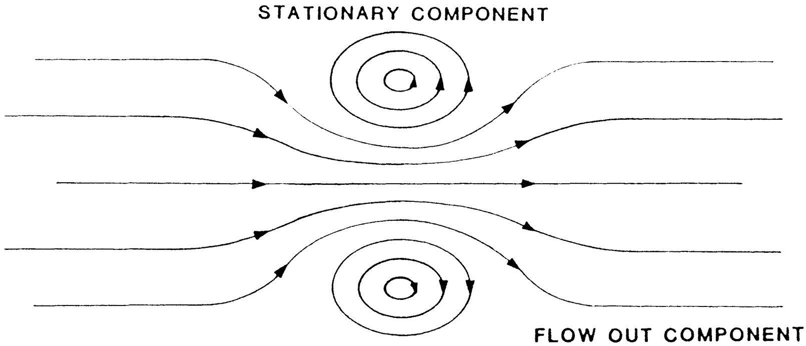


Fig. 5. Schematically depicted interacting feature of the flow out component and the stationary component of gas which is making eddy motion.

$$| \mathrm{rot}(\boldsymbol{V} \times \boldsymbol{\omega}) | / | \nu \nabla^2 \boldsymbol{\omega} |= R, \tag{39}$$

where

$$R = \rho r_e V_{ro} / \eta. \tag{40}$$

In this expression of the Raynolds number R, η, and ρ denote viscosity of the fluid and the density of the fluid, respectively. Because r_e has the order of 1AU, the Raynolds number R becomes $10^9 \sim 10^{12}$ for the flow velocity of 1km/s∼10km/s. This means that the eddy can not be locally maintained (because $\partial \boldsymbol{\omega} / \partial t = \mathrm{rot}(\boldsymbol{V} \times \boldsymbol{\omega})$, it flows with fluid) to stay in the flowing gas under the condition of the regular viscosity.

As has been already been assumed in the previous paper (Oya, in this issue), the most important effect may be brought by the interaction with the magnetic field. The flowing plasma makes crash with the stationary component of the gas and the ionization takes place due to effect of heating and also critical ionization. The ionized component then trapped by the troidal and poloidal components of the magnetic field. Poloidal components come from the region of the proto-sun where intense dynamo effects are taking place. In the meteorite, the magnetic field is confined with intensity of the order of Oe (Nagata, in this issue). If we use this value as a key value to represent the magnetic field in the partially ionized disc that gives the magnetic field $B = B_0 (r_0 / r)^2$, in the proto solar nebula in the region from 1 to 2AU, we can expect the magnetic field in a range about 0.04 to 0.16 gauss at the position of proto-Jupiter and 10^{-3}gauss to 4×10^{-3}gauss at the position of proto-Neptune. Corresponding to this value we can find the proton cyclotron frequency given in Table I. This poloidal component may contribute to make stationary components making microscopic rotation of the ionized component of the fluid (see Fig. 6).

Contrary to this intense component, there may another weak troidal components of the magnetic field in the disc region that is locally generated due to the filament currents in the azimuthal direction in the region where the flowing gas makes collision with the stationary components. For the magnetic field with the order of nT, protons make gyration with frequency

Table I. Proton cyclotron frequency at corresponding points for weak and intense cases of the assumed magnetic field in the proto-solar system.

	Weak case	Intense case
Jupiter	60 Hz	2.43×10^2 Hz
Saturn	18 Hz	72 Hz
Uranus	4.3 Hz	17.2 Hz
Neptune	1.52 Hz	6.08 Hz

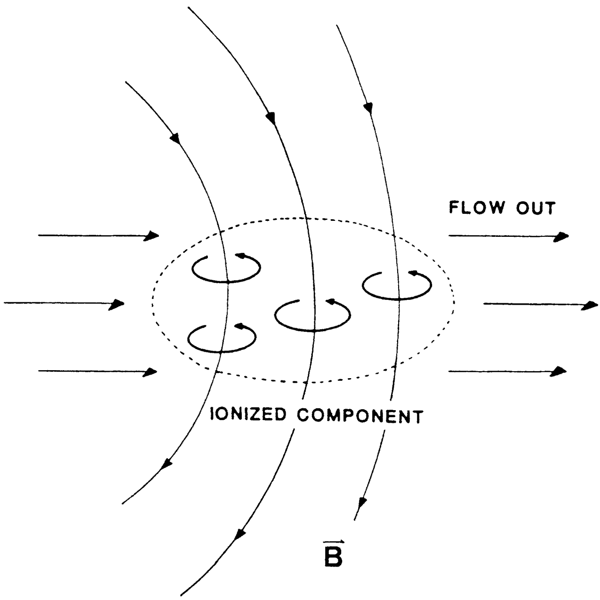


Fig. 6. Magnetic field effect on the locally ionized gas for formation of the large scale eddy.

of 0.015Hz and with Larmar radius of the order of 50km. This mean that the solar wind in this disc still severely affecting to sweep out the stationary component before completion of the eddy motion.

Conclusion

Computer simulation have been carried out for the gas accumulation of the material to form the seeds of the giant planets, in the stage of the bi-polar flow where the gas is flowing out with constant speed in a relatively thin disc. The processes of accumulations of material under this flowing out gas condition have already been predicted by a quasi-linear theory given by previous paper (Oya, 1982; see also Oya in this issue). Because the quasi-linear theory is based on the linearized stage of the essentially nonlinear equations, numerical simulation is needed to confirm the processes of the density waves associated with the self gravity; the waves makes coupling with the flowing gas components and make trapping of the flowing material resonance with the self gravity field waves.

In this stage, the simulations have been made for the same model that has been studied in the quasi-linear theory, by solving Euler equation of fluids for the thin disc where the gas is flowing out with constant velocity. The Poisson's equations for the self gravity potential have also been considered with conservation of the gas flow. For the normalized series of differential equations, the calculations have been made applying the two step Lax-Wendroff method. In whole procedure of this studies, only the radial and vertical variations of the fluid parameters and gravity fields are considered applying ring distribution model of the gas.

Three cases of the conditions have been studied. In the first and second cases, the numerical value of the initial perturbation is set to be 10^{-3} of the background density; the initial perturbation is set so as to show local maximum at each position of each proto-planet. The results show that the stationary component makes growth with speed to be twice of the background density level within 3yrs for the condition of the flow out velocity of 5km/s $\sim$ 10km/s with the background density of $10^{12}/\mathrm{cm}^3$ (the first case) and $2 \times 10^{11}/\mathrm{cm}^3$ (the second case) at the position of proto-Jupiter. Though the stationary components show initially a slight movement, the components become finally stationary and clearly show that each peak point of the fluctuation clearly satisfy Titius-Bode's law; *i.e.*, we can consider that the accumulated material is possibly the origin of giant planets.

To check uniqueness of the processes to accumulate the source material of the giant planets, different initial condition is given in the third case of the simulation where the initial perturbation has been given as the summation of many wave numbers together. The results show again that the stationary components make growth at points that fit to the series of positions to satisfy the Titius-Bode's law.

All of the simulation results thus give confirmation to the processes of accumulation of the material by the density wave where the local gravity potential makes trapping the flowing material.

Interaction of the flowing gas with stationary component takes place with rate of $1 \sim 10/s$ at the position of Jupiter and 3×10^{-2} to $3 \times 10^{-1}/s$ at the position of Neptune. To keep the stationary component remaining in the flowing out material in the nonlinear stage, the effects of the magnetic field is important. The possible magnetic field intensity in the early stage of the bi-polar flow gives the proton cyclotron frequency in a range from 60Hz to 240Hz at the position of Proto-Jupiter and 1.5 to 6Hz at the position of Neptune. The value shows that the gas can be trapped to the magnetic field when the gas is ionized. The ionization of gas is naturally accepted in region of the strong interaction of the flowing-out components with the stationary components. To have more realistic understanding of the evolution of the accumulation processes of the material for giant planets, considerations are needed for the studies on the instabilities in the azimuthal components. These works are deferred for future studies as has already been pointed out in the quasi-linear paper given in this issue.

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