

Deformation of Porous Ice-Rock Mixtures and an Application to the Densification of Icy Satellites

Norikazu Maeno, Masahiko Arakawa, and J. Leliwa-Kopystynski^{*)}

*Institute of Low Temperature Science, Hokkaido University,
Sapporo 060, Japan*

**)Institute of Geophysics, Warsaw University,
Pasteura 7, 02-093 Warsaw, Poland*

1. Introduction

The recent Voyager 1 and 2 missions to the outer giant planets have brought an enormous amount of new information about the solar system (Smith *et al.*, 1979; 1981; 1982; 1989). The discoveries and detailed information on many icy satellites of Jupiter, Saturn, and Neptune had great impacts on the field of glaciology. This field of study had only examined ice and snow appearing on the Earth traditionally and most investigations had been carried out in a narrow range near the melting point of ice and atmospheric pressure. Now it has become evident that in the ‘cosmoglaciology’ or ‘planetary glaciology’, which deals with all the forms of snow and ice in space, physical properties of various ices including high-pressure phases and clathrates are required, especially at low temperatures. It is also obvious that most ices studied in cosmoglaciology occur as mixtures of various rocks and that the information on the physical properties of ice-rock mixtures is also of most importance. However, such data are very scarce. This paper gives the result of our densification experiment on porous ice-rock mixtures, together with the reported results of deformation experiments on solid ice-rock mixtures, and its application to the numerical calculation of

densification and thermal evolution of icy satellites of Saturn.

2. Relative viscosity of ice-rock mixtures

The deformation properties of ice-rock mixtures may be different from those of pure ice depending on the kind, content, and size of rocks contained (Friedson and Stevenson, 1983). An increase in the hardness and viscosity is generally expected since hard rock particles dispersed in ice impede the movement of dislocations, the glide between ice grains and the molecular diffusion relevant to mechanical deformation. At temperatures near the melting point of ice, however, a decrease in the viscosity may be expected due to regelation (or pressure melting) and chemical impurities involved in rock particles.

If the ice-rock mixture is assumed to be a dilute suspension composed of a viscous fluid containing a small amount of fine solid spheres, its relative viscosity (η) is given theoretically by Einstein (1906) as

$$\eta = \eta_f / \eta_0 = 1 + 2.5f, \quad (1)$$

where η_f and η_0 are the viscosity coefficients of the ice-rock mixture and the ice, respectively, and f is the volume fraction of rock. Equation (1) is valid only for f smaller than 0.2. For higher volume fractions (less than 0.6) and non-Newtonian fluids, Thomas (1965) obtained the following empirical relation,

$$\eta = 1 + 2.5f + 10.05f^2 + 0.00273 \exp(16.6f). \quad (2)$$

It is also known that the relative viscosity of a suspension of a single particle size is smaller than that of a size-distributed suspension. Roscoe (1952) showed that its lower limit is given by

$$\eta = (1 - f)^{-2.5}. \quad (3)$$

The lower and upper bounds of relative viscosity predicted by Eqs. (1) to (3) are shown in Fig. 1, together with some experimental results, most of which were obtained to estimate the creep rate in the bottom layer of glaciers. Though different deformation modes and techniques were used in each experimental study, the effective viscosity was defined as the steady-state strain rate divided by the applied stress to estimate relative viscosities.

In their tension experiments of ice containing fine silica particles of about 15 nm, Nayar *et al.* (1971) found a drastic increase in the relative viscosity, which amounted to 30 at the volume fraction of 0.01 at -4.8 and -11.9°C . According to the compression measurement made around -9°C by Hooke *et al.* (1972), the relative viscosity of ice containing sands (mean diameter $180\mu\text{m}$) increased exponentially with the volume fraction of sand, but when the fraction was low the relative viscosity scattered and became

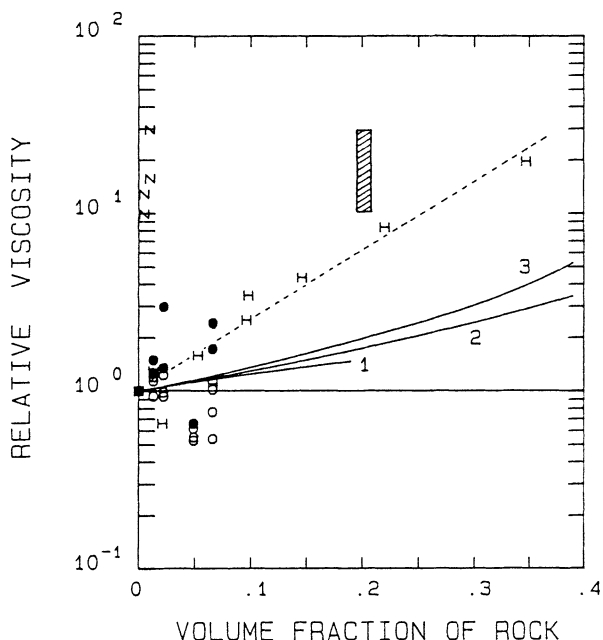


Fig. 1. Relative viscosity of ice-rock mixtures. H: Hooke *et al.* (1972), N: Nayar *et al.* (1971), Circles: Baker and Gerberich (1979), open and solid circles denote values obtained at temperatures higher and lower than -20°C , respectively. Lines 1, 2, and 3 correspond to Einstein (1906), Roscoe (1952), and Thomas (1965), respectively. The range of relative viscosity obtained in the present experiment of porous ice-rock mixture is represented by a box.

even smaller than unity in some cases, *e.g.*, $f = 0.021$ in Fig. 1. Similar compression experiments of low rock fractions were made by Baker and Gerberich (1979) at temperatures from -5 to -40°C . Their results indicated that the relative viscosity of ice containing sieved Ottawa sands (diameters $106 \sim 125 \mu\text{m}$ and volume fractions less than 0.1) is smaller than unity at temperatures higher than about -20°C but larger than unity at lower temperatures. They attributed the former's decrease to the grain growth and grain-boundary sliding and the latter's increase to the increase in the activation energy and internal stress created by sand inclusions. According to the recent report by Budd and Jacka (1989), sandy ice samples at low stress give practically a constant relative viscosity even if the sand volume fraction is changed from 0.004 to 0.15. Durham and Kirby (1990), however, reported that the mixtures of ice and silica particulates at volume fractions above 0.3 are many orders of magnitude more viscous at lower temperatures, below 200 K. The relative viscosity, at 160 K, estimated from the data in Fig. 1 of their paper, amounts to roughly 9 and 4500 at the volume fractions of 0.3 and 0.56, respectively.

All the above results are not correlated but lead to the general conclusion that the relative viscosity of an ice-rock mixture increases with the rock content at a volume fraction higher than about 0.1; on the other hand, at a lower rock content, the effects of grain growth of ice and the grain-boundary sliding enhanced by chemical impurities involved become relatively important in comparison to those of dislocation motion and molecular diffusion, depending on temperature.

We have discussed above the deformation of solid ice-rock mixtures. When we consider the internal structure and deformation of small icy planetary bodies we need to add the information concerning the relative viscosity of porous ice-rock mixtures, which has not been measured so far. We performed recently a densification experiment of porous ice-rock mixtures, which will be given in the next section. The obtained result of relative viscosity is shown in Fig. 1. It is apparent that the relative viscosity of porous ice-rock mixture is much larger than that of a solid ice-rock mixture.

3. Deformation of porous ice-rock mixtures

Densification experiments of porous ice-rock mixtures were carried out

by unidirectional compression of samples contained in stainless steel cylinders. The cross-sectional areas of the cylinders were $(3.2 \sim 12.6) \times 10^{-4} \text{m}^2$, and the pressure was applied by putting weights or oil press, in a range of effective pressures from 2.25 to 17.7 MPa. Measurements were conducted at temperatures ranging from -11 to -150°C ; only the results obtained at -25°C are reported here. More details of the experimental procedures are found elsewhere (Leliwa-Kopystynski *et al.*, 1989; Leliwa-Kopystynski and Maeno, in preparation).

Samples of ice-rock mixtures were prepared by mixing ice and rock particles at -10°C . Ice particles were formed by freezing small water droplets on an aluminum plate cooled at -10°C or by disintegrating and sieving pure natural snow blocks collected at the Nakayama Path in the suburb of Sapporo and stored in a cold room at -15°C for a year. The mean diameter of ice particles used was $200 \mu\text{m}$ or less. Several kinds of rock particles were used, but only the result for the ice plus garnet lherzolite is given in this paper. The garnet lherzolite was found in the form of a small boudin ($1.5 \text{ m} \times 2.5 \text{ m}$) within granulites of the Gory Sowie Mountains gneiss block near Bystrzyca Gorna in the Polish Sudetes. Its density was 3060 kg/m^3 and the diameters of crushed particles measured from $10 \mu\text{m}$ to 3 mm .

The densification of the mixture sample was studied by measuring the change of height and interpreting it as the porosity. The porosity (q) of a sample is written as

$$q = (\text{volume of pores})/(\text{volume of sample}) = 1 - \rho/\rho_o, \quad (4)$$

where ρ is the bulk density of the sample and ρ_o is a constant determined by the densities of ice (ρ_I) and rock (ρ_R) and the mass fraction of rock (c), described as follows:

$$\rho_o = 1/[(1 - c)/\rho_I + c/\rho_R]. \quad (5)$$

When no rocks are contained in a sample, that is, $c = 0$, then ρ_o is identical to ρ_I . The relative density is defined as the bulk density normalized by ρ_o . The relation between the volume (f) and mass (c) fractions of rocks is

$$f = (1 - q)[1 - (1 - c)\rho_o/\rho_I]. \quad (6)$$

Figure 2 shows the rate of pore decrease, $-(dq/dt)/q$, plotted against the applied stress at three porosities. A power relation, $-(dq/dt)/q \propto \tau^n$, is found at each porosity with $n = 3.72 \sim 4.20$ for $c = 0$ and $n = 4.26 \sim 4.46$ for $c = 0.5$. As the steady-state creep of pure polycrystalline ice has been shown to be dominated by the dislocation mechanism with n ranging from 3 to 5, the above result suggests that the dislocation mechanism also plays a most important role in the densification of porous ice-rock mixtures.

The relative viscosity of the porous ice-rock mixture was estimated by the following equation,

$$\eta = \eta_f/\eta_0 = [-(dq/dt)/q]_{\text{pure ice}}/[-(dq/dt)/q]_{\text{ice-rock mixture}}. \quad (7)$$

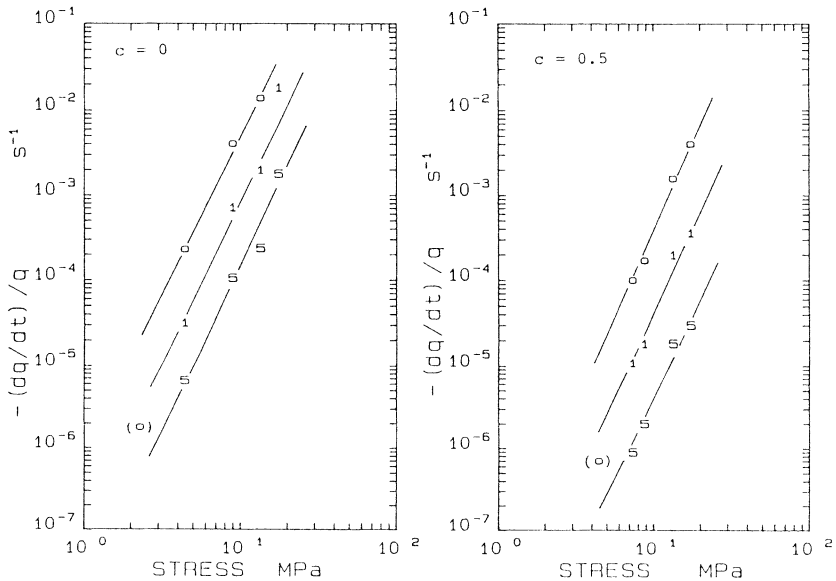


Fig. 2. Relations between the pore decreasing rate and stress. Numerals 0, 1, and 5 denote porosities of $q = 0.15$, 0.1 , and 0.05 , respectively. Temperature is -25°C and the mass fractions of rock are $c = 0$ (left) and 0.5 (right).

The range of the relative viscosity obtained is represented as a box in Fig. 1 and we find its values ranged from 9 to 28 in a porosity range from 0.1 to 0.15. The mass fraction, $c = 0.5$, corresponds to the volume fraction $f = 0.196 \sim 0.207$ in the porosity range. It is important to note that the relative viscosity of porous ice-rock mixtures is larger than that of solid ice-rock mixtures. The physical mechanism of the increase is not clear at present and requires of more systematic measurements of the densification process of porous ice-rock mixtures.

4. Densification histories of Saturn's icy satellites

We present here some examples of numerical calculations of the densification and thermal evolutions of Saturn's icy satellites; more details are found elsewhere (Arakawa and Maeno, in preparation). The numerical constants used in the calculation are listed in Table I. The impact melting during the accretion period of the icy satellites is not considered here as a possible densification mechanism because the impact velocities of icy planetesimals are not large enough to cause a shock melt in the case of Saturn's icy satellites. We assumed that the densification of icy satellites proceeds solely by the pressure-sintering of porous ice-rock mixtures. The initial distribution of pores in the satellites was assumed to be uniform and the initial porosity was put as 0.4, which corresponds to the first maximum density of snow attained by mechanical compaction (Maeno and Ebinuma, 1983).

The ambient temperature around the icy satellites was put to be 80 K, and the initial temperature profile in each satellite was estimated by a calculation according to the method of Mizutani *et al.* (1972); the temperature profiles obtained by the calculation during the accretion time of 1×10^5 yrs are almost identical to those of the cold thermal evolution model of Ellsworth and Schubert (1983).

The spherical symmetric heat conduction equation with a heat production term due to radioactive elements was numerically solved at each distance from the center and time, simultaneously with the pressure-sintering equations of densification caused by dislocation, lattice-diffusion, and boundary-diffusion mechanisms (Maeno and Ebinuma, 1983). We applied the relative viscosity of porous ice-rock mixtures obtained above by putting that

Table I. Physical constants used in the calculation.

Abundances of radioactive isotopes ($\times 10^{-8}$ kg/kg)

<i>Elements</i>	<i>Abundance</i>
^{238}U	1.092
^{235}U	0.0079
^{232}Th	4.4
^{40}K	9.52

Thermal conductivity (W/m/K)

Ice: $K_i = 488.19/T + 0.4685$

Rock: $K_r = 418.4/(30.6 + 0.21T)$ ($T \leq 500\text{K}$)

Heat capacity (J/kg/K)

Ice: $C_{pi} = 2117 - 7.8(273 - T)$

Rock: $C_{pr} = 0.273(\Theta/T)^2 - 5.6(\Theta/T) + 30.58$

Θ : Debye temperature of olivine=760 K

Density (kg/m^3)

Ice: $\rho_I = 917$

Rock: $\rho_R = 3260$

the relative viscosity of each densification mechanism is 20, that is, the total densification rate is reduced by a factor of 20 as compared to that of pure ice. The steps of time and distance in the calculation by the Crank-Nicholson implicit finite difference scheme were 2.5×10^5 yrs and 10 km, respectively. In the calculation the effective thermal conductivity of porous ice-rock mixtures and the growth of ice grains were also taken into account.

It was shown by trial calculations that in Saturn's icy satellites the highest temperature attained in the satellites was below 200 K and never gives rise to a differentiation by melting, and that the most probable range of the initial radius of ice grains might be from 0.1 to 10 μm . The present internal structure of Mimas thus calculated, allowed us to estimate the

moment of inertia and compare it with that accurately obtained by Dermott and Thomas (1988) using Voyager images. Their calculations deduced the mean radius at 198.8 km, the mean density at 1137 kg/m^3 and the moment of inertia at 0.353. Through many calculations with probable combinations of initial porosity and grain radius, it was revealed that the low moment of inertia could only be explained by the combination of an initial porosity of 0.46 and the initial ice grain radius of $1 \mu\text{m}$.

Figure 3 gives the density and temperature profiles of Mimas calculated with an initial ice particle radius of $1 \mu\text{m}$ and porosity of 0.46. It was shown that the densification proceeds radially from the center in an outward direction and its rate increases sharply when the temperature becomes higher than about 100 K at the time of about 2×10^7 yrs. The increase in the density nearly ceased at about 2×10^8 yrs, since the temperature reached a maximum at about 1×10^8 yrs and thereafter only decreased toward the radiative equilibrium temperature at the surface, 80 K. The growth of ice grains was noted within the radius of 70 km, but its rate was small, their radii became only 2 to 3 times larger in the calculation period and giving no appreciable reduction in the densification rate.

The final (4.5×10^9 yrs) density profile calculated suggests a two-layer

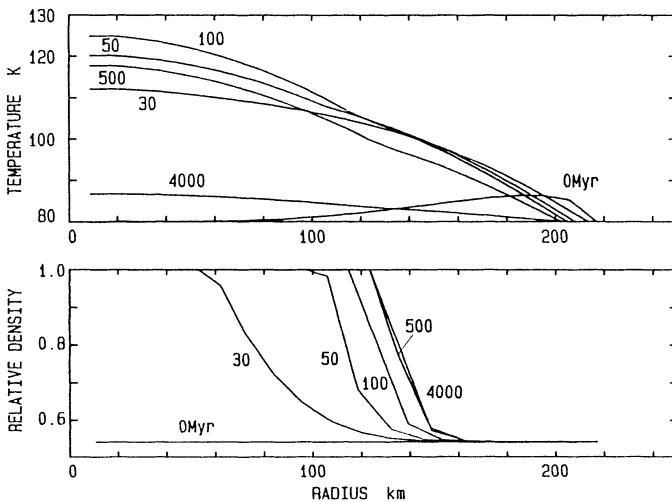


Fig. 3. Thermal and densification history of Mimas. The initial porosity and grain radius of ice are 0.46 and $1 \mu\text{m}$, respectively.

structure, that is, the present structure of Mimas is composed of a solid ice-rock core (roughly 135km in radius), a narrow transition layer (roughly 15 km), and a porous ice-rock mantle (roughly 50 km). Similar results of densification and thermal processes were obtained in the case of Enceladus; the size was very similar to that of Mimas.

Next, we give the calculation results of other icy satellites. The obtained densification and thermal evolutions of the larger satellites (larger radii than roughly 500 km), Rhea, Iapetus, Dione, and Tethys, are similar in their general features; only the case of Rhea is shown in Fig. 4. It is apparent that owing to the initial accretional heating effect the densification proceeds from the surface toward the center and the densification of the whole body is completed by 3×10^8 yrs. This mode of densification is very characteristic and might cause some interesting tectonic variations. As the densification proceeds, a thin solid crust is formed at the surface layer, which then will be broken due to its shrinkage and produce extensive faults and other deformed structures. It is reasonable to assume that some of these features might remain till the present as troughs, ridges, and others frequently found on surfaces of Rhea, Dione, and Tethys (Smith *et al.*, 1982).

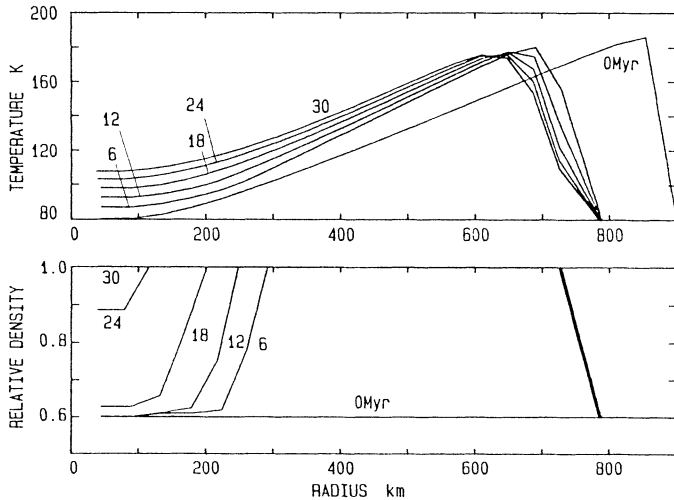


Fig. 4. Thermal and densification history of Rhea. The initial porosity and grain radius of ice are 0.40 and $1 \mu\text{m}$, respectively.

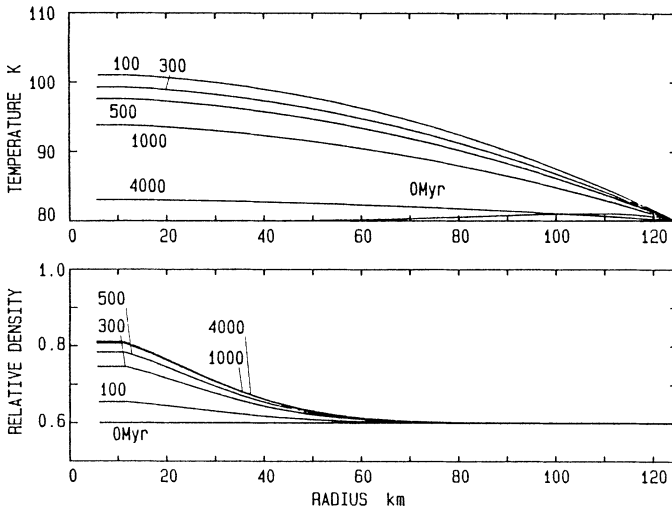


Fig. 5. Thermal and densification history of Phoebe. The initial porosity and grain radius of ice are 0.40 and $1 \mu\text{m}$, respectively.

It is shown in Fig. 5 that in the interior of Phoebe, the smallest satellite studied in this calculation, the densification proceeds very slowly and the relative density at the center only becomes about 0.8 in 4.5×10^9 yrs. The slow evolution is attributed to the small internal pressure and low temperature. Similar results are expected in the densification and thermal evolutions of other small satellites, Prometheus, Pandora, Janus, Epimetheus, and Hyperion.

Acknowledgements

The authors would like to thank Mr. S. Murakami of the Kyocera Company and Dr. Y. Mizuno of the Institute of Low Temperature Science for their help in the densification experiment of ice-rock mixtures, and Prof. H. Mizutani of the Institute of Space and Astronautical Science for his critical reading of the manuscript. They are also indebted to Dr. N. Bakun-Czubarow of Institute of Geological Sciences, Polish Academy of Sciences, for providing and analyzing the rock sample, garnet lherzolite. The study was supported partly by the Grant-in-Aid for Scientific Research on Priority Areas (Origin of the Solar System) of Japanese Ministry of Education, Science, and Culture (Nos. 6261005, 63611005, and 01611008). One

of the authors (J. L.-K.) acknowledges the Japan Society for the Promotion of Science for making it possible for him to study in Hokkaido University.

References

- Arakawa, M. and N. Maeno, Internal structures of Saturn's icy satellites: Densification and thermal evolution analysis, (in preparation).
- Baker, R.W. and W.W. Gerberich, 1979, The effects of crystal size and dispersed-solid inclusions on the activation energy for creep of ice, *J. Glaciology*, **24**(90), 179.
- Budd, W.F. and T.H. Jacka, 1989, A review of ice rheology for ice sheet modelling, *Cold Regions Sci. Tech.*, **16**, 107.
- Dermott, S.F. and P.C. Thomas, 1988, The shape and internal structure of Mimas, *Icarus*, **73**, 25.
- Durham, W.B. and S.H. Kirby, 1990, Planetary ices: A comparison of rheologies at $T < 200$ K, *Lunar Planet. Sci. Conf.* **XXI**, 305.
- Einstein, A., 1906, Eine neue Bestimmung der Molekuldimensionen, *Ann. Phys.*, **19**, 286.
- Ellsworth, K. and G. Schubert, 1983, Saturn's icy satellites: thermal and structural models, *Icarus*, **54**, 490.
- Eluszkiewicz, J., 1990, Compaction and internal structure of Mimas, *Icarus*, **84**, 215.
- Eluszkiewicz, J. and J. Leliwa-Kopystynski, 1989, Compression effects in rock-ice mixtures: an application to the study of satellites, *Phys. Earth Planet. Int.*, **55**, 387.
- Friedson, A.J. and D.J. Stevenson, 1984, Viscosity of rock-ice mixtures and applications to the evolution of icy satellites, *Icarus*, **56**, 1.
- Hooke, R. Leeb., B.B. Dahlin, and M.T. Kauper, 1972, Creep of ice containing dispersed fine sand, *J. Glaciology*, **11**(63), 327.
- Leliwa-Kopystynski, J., S. Murakami, Y. Mizuno, N. Maeno, and N. Bakun-Czubarow, 1990, Compaction of icy/rocky mixtures at low temperatures (from 262 to 77 K): preliminary results, in *High Pressure Research*, Vol. 5 (Proc. 12 AIRAPT and 27 EHPRG Conference, Paderborn, 1989), Gordon and Breach Sci. Pub., p. 696.
- Leliwa-Kopystynski, J. and N. Maeno, Icy/rocky mixtures: compaction experiments and interpretation (in preparation).
- Maeno, N. and T. Ebinuma, 1983, Pressure sintering of ice and its implica-

- tions to the densification of snow at polar glaciers and ice sheets, *J. Phys. Chem.*, **87**, 4103.
- Mizutani, H., T. Matsui, and H. Takeuchi, 1972, Accretion process of the Moon, *The Moon*, **4**, 658.
- Nayar, H.S., F.V. Lenel, and G.S. Ansell, 1971, Creep of dispersions of ultrafine amorphous silica in ice, *J. Appl. Phys.*, **42**(10), 3766.
- Roscoe, R., 1952, The viscosity of suspensions of rigid spheres, *Brit. J. Appl. Phys.*, **3**, 26.
- Smith, B.A. and 21 authors, 1979, The Jupiter system through the eyes of Voyager 1, *Science*, **204**, 951.
- Smith, B.A. and 26 authors, 1981, Encounter with Saturn : Voyager 1 imaging science results, *Science*, **212**, 163.
- Smith, B.A. and 28 authors, 1982, A new look at the Saturn system: The Voyager 2 images, *Science*, **215**, 504.
- Smith, B.A. and 64 authors, 1989, Voyager 2 at Neptune: Imaging science results, *Science*, **246**, 1422.
- Thomas, D.G., 1965, Transport characteristics of suspensions, *J. Colloid Sci.*, **20**, 267.